FROM Last Time

Q5 There are 10 questions on the next DM2 exam. If the total score is 100 and each question is at least 5 points, how possible ways are there to assign the points on the questions?

We add 50 points — automatically 5 to each question. Then we must distribute the remaining 50 points among 10 questions. each point corresponds to a star, and we can separate the 10 questions with 9 bars:

Q1   Q2 Q_3 Q_{10}

* * * | * * ... | ~~ | | | * ...

We must count the # of configurations with 50 stars & 9 bars:

→ 59 slots → choose 9 of them for the bars.

Ans $\binom{59}{9} = \boxed{12,565,671,261}$

$$\frac{(59)(58)(57)\cdots(51)}{9\cdot 8\cdot 7 \cdots 1}$$

Next example: There are 6 different kinds of desserts in your refrigerator (lots of each). You have to decide which dessert to eat on which weekday (MTWRF). How many different ways can we choose the 5 desserts for a week's sugar infusion?

- choc cake
- choc. pudding
- choc. ice cream
- cheesecake with choc drizzle
- a bar of milk chocolate
- a bar of dark chocolate

6^5 ← if we were keeping track of what day each dessert is eaten.

We are counting the sets of 5 desserts (repetition allowed)

choc. cake | choc pudding | choc ice cream | cheesecake choc drizzle | bar milk choc | bar dark chocolate

* * * * *

5 bars separating 6 desserts

5 stars ← days of week

$$\text{ans } \binom{10}{5} = \frac{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}\cdot 7\cdot\cancel{6}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = \boxed{252}$$

Recursion Relations

A recursion formula is a way to generate a sequence $(\underline{a_1}, \underline{a_2}, \underline{a_3}, \ldots)$:

1. Usually, we are given initial conditions

$a_1 = \_\_\_$

$a_2 = \_\_\_$

② Given a recursion formula: a formula used to calculate the next item in the sequence. (usually in terms of the previous items.)

$$a_n = \sim a_{n-1} + \sim a_{n-2} \sim$$

Our job: ① figure out what the sequence is

② What properties does the sequence have?

③ (Sometimes) We can find a closed-form formula for the sequence:

$$a_n = \sim$$

only depends on n, not a_{n-1} or $a_{n-2}, \ldots$

Example ① Let $F_1 = 1$, $F_2 = 1$,

$$F_n = F_{n-1}$$

$$(F_n) = (1, 1, 1, 1, 1, \ldots$$

② Let $F_1 = 1$, $F_2 = 1$,

$$F_n = F_{n-1} + F_{n-2}$$

$(F_n) = (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots)$

Fibonacci sequence

Fun Friday Quiz

① What is your favorite chocolate dessert?

② (A) How many 2 person committee could we select from the third row of this class? (10 people)

(B) If $f: A \to B$ is a functions then $f^{-1}(\{b\}) = \boxed{}$ for each $b \in B$

(C) How many different 3 digits numbers are there where the last digit is a prime digit?