

FROM Last Time

(Q5) There are 10 questions on the next DM2 exam. If the total score is 100 and each question is at least 5 points, how many possible ways are there to assign the points on the questions?

We add 50 points — automatically 5 to each question. Then we must distribute the remaining 50 points among 10 questions. Each point corresponds to a star, and we can separate the 10 questions with 9 bars:

Q₁ Q₂ Q₃ ... Q₁₀
* * * | * * .. | * * .. | * ..

We must count the # of configurations with 50 stars & 9 bars.

→ 59 slots → choose 9 of them for the bars.

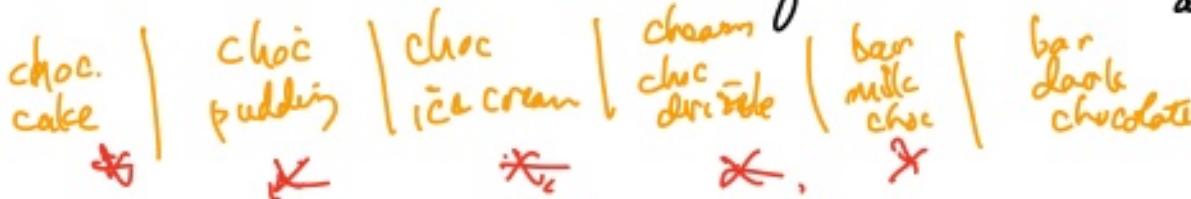
Ans $\binom{59}{9} = \frac{(59)(58)(57)\dots(51)}{9 \cdot 8 \cdot 7 \dots 1}$

Next example: There are 6 different kinds of deserts in your refrigerator (lots of each). You have to decide which desert to eat on which weekday (MTWRF). How many different ways can we choose the 5 deserts for a week's sugar infusion?

- choc cake
- choc. pudding
- choc. ice cream
- cheesecake with choc drizzle
- a bar of milk chocolate
- a bar of dark chocolate.

$6^5 \leftarrow$ if we were keeping track of what day each dessert is eaten.

We are counting the sets of 5 deserts (repetition allowed)



5 bars separating 6 desserts

5 stars \leftarrow days of week

$$\text{ans } \binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \boxed{252}$$

Recursion Relations

A recursion formula is a way to generate a sequence $(\underline{a_1}, \underline{a_2}, \underline{a_3}, \dots)$:

① Usually, we are given initial conditions

$$a_1 = \underline{\quad}$$

$$a_2 = \underline{\quad}$$

② Given a recursion formula: a formula used to calculate the next item in the sequence.
(usually in terms of the previous terms.)

$$a_n = M a_{n-1} + 3 a_{n-2}$$

Our job: ① Figure out what the sequence is

② What properties does the sequence have?

③ (Sometimes) We can find a closed-form formula for the sequence:

$$a_n = \cancel{\text{some formula}}$$

↑
only depends on n ,
not a_{n-1} or a_{n-2}, \dots

Example ① Let $F_1 = 1, F_2 = 1,$

$$F_n = F_{n-1}$$

• $(F_n) = (1, 1, 1, 1, 1, \dots)$

② Let $F_1 = 1, F_2 = 1,$

$$F_n = F_{n-1} + F_{n-2}$$

$$\langle F_n \rangle = (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \\ \underbrace{\text{Fibonacci sequence}}_{89, \dots})$$

Fun Friday Quiz

① What is your favorite chocolate dessert?

② A How many 2 person committee could we select from the third row of this class?
(10 people)

B If $f: A \rightarrow B$ is a function
then $f^{-1}(\{b\}) =$ []
for each $b \in B$

C How many different 3 digit numbers are there where the last digit is a prime digit?